The peak in the nonlinear ac resistivity of granular superconductors

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Abstract

We model $s$-wave and $d$-wave disordered granular superconductors with a three-dimensional random network of Josephson junctions with finite self-inductance. The nonlinear ac resistivity $\rho_2$ was calculated numerically. We find a peak in $\rho_2$ as a function of temperature, in good agreement with recent experiments. The value of $\rho_2$ at the peak temperature $T_p$ depends on the current amplitude $I_0$ as a power law, $\rho_2(T_p) \sim I_0^\alpha$. We find that $\alpha$ depends on the self-inductance and current regimes. In the weak current regime is $\alpha = 0.5 \pm 0.1$ and independent of the self-inductance for both of $s$- and $d$-wave materials. In the strong current regime, $\alpha$ depends on the screening, with $\alpha \approx 1$ for some interval of inductance in agreement with measurements in $d$-wave high $T_c$ ceramic superconductors.

Key words: granular superconductors; pi junctions; d-wave superconductivity; Josephson networks

Recently, Yamao et al.\textsuperscript{[1]} have measured the ac linear resistivity $\rho_0$ and the nonlinear resistivity $\rho_2$ of ceramic superconductor YBa$_2$Cu$_3$O$_y$. $\rho_2$ is defined as the third coefficient of the expansion of the voltage $V(t)$ in terms of the external current $I_{ext}$ as $V = \rho_0 I_{ext} + \rho_2 I_{ext}^3 + \ldots$. When the sample is driven by an ac current $I_{ext}(t) = I_0 \sin(\omega t)$, one can obtain $\rho_2$ from

$$\rho_2 = -\frac{4V_3'}{I_0^3}, \quad V_3' = \frac{1}{2\pi} \int_{-\pi}^{\pi} V(t) \sin(3\omega t) d(\omega t).$$  \hspace{1cm} (1)

Yamao et al. have found that $\rho_2$ has a maximum value at a temperature $T_p$ near the intergrain ordering temperature of their sample. They observed that $\rho_2$ depends with $I_0$ as $\rho_2(T_p) \sim I_0^\alpha$, with $\alpha \approx 1.1$.

It is now believed that the gap of high-$T_c$ superconductors has $d$-wave symmetry. This makes possible to have weak links with negative Josephson coupling between the superconducting grains in high-$T_c$ ceramics, which are called $\pi$-junctions [2]. Therefore, they can be modeled with a network of Josephson junctions with random couplings, given by the hamiltonian [3–5]

\[ H = -\sum_{<ij>} J_{ij} \cos(\theta_i - \theta_j - A_{ij}) + \frac{1}{2L} \sum_{p} \Phi_p^2. \]  \hspace{1cm} (2)

Here $\theta_i$ is the superconducting phase of the grain at the $i$-th site of a cubic lattice, $J_{ij}$ is the Josephson coupling between grains, and $L$ is the self-inductance of a loop (mutual inductances are neglected). The first sum is taken over all nearest-neighbor pairs and the second sum is taken over all elementary plaquettes on the lattice. The total magnetic flux threading through the $p$-th plaquette is $\Phi_p = \frac{\Phi_0}{2\pi} \sum_{<ij>} A_{ij}$ with $A_{ij} = \frac{2\pi}{\Phi_0} \int_{<ij>} A(\mathbf{r}) d\mathbf{r}$. We model the $d$-wave superconducting case by taking $J_{ij}$ as a random variable equal to $J$ or $-J$ with equal probability (representing 0 and $\pi$ junctions respectively), and also the $s$-wave superconducting case by taking $J_{ij} > 0$ and uniformly distributed in $[0,2J]$. The effect of screening currents is characterized by the dimensionless inductance $\tilde{L} = (2\pi/\Phi_0)^2 L J$. The $d$-wave model has been able to reproduce the paramagnetic Meissner effect [3] observed experimentally in ceramic high-$T_c$ ceramics [6]. Kawamura [4,5] proposed that there is a chiral glass phase, which has been seen experimentally in the nonlinear ac magnetic susceptibility [7] and in the aging phenomenon [8].

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To calculate transport properties we use the resistively shunted junction model in which the dissipative ohmic current due to an intergrain resistance $R$ and the temperature dependent Langevin noise current are added to the Josephson current [3]. This leads to a set of dynamical equations for $\theta_i$ and $A_{ij}$ [3], which are solved numerically for a given temperature $T$ and driving current $I_{ext} = I_0 (\sin \omega t)$ [9,10]. The temperature dependence of the nonlinear resistivity $\rho_2$ for different values of $I_0$ is shown in Fig. 1 for the $s$-wave system (upper panel) and for the $d$-wave system (lower panel) for $L = 1, \omega = 0.001$ and $8 \times 8 \times 8$ samples.

Fig. 1. Temperature dependence of the nonlinear resistivity $\rho_2 \propto V_{j}^{2}/I_{0}^{2}$ for the $s$-wave (upper panel) and the $d$-wave case (lower panel) for $L = 1, \omega = 0.001$ and $8 \times 8 \times 8$ samples.

 dependently on $L$, as it is shown in Fig. 2.

In conclusion, we have calculated the non-linear ac resistivity exponent $\alpha$ for $s$ and $d$-wave granular superconductors, obtaining two distinct current regimes. For weak currents $\alpha$ is independent of the screening strength and of types of pairing symmetry, while in the opposite case this exponent depends on $L$. Since real current is $I = \frac{2e}{h}I_0$, and $J \sim 10^2 K$, then for $I_0 \sim 0.1$ we have $I \sim 10^{-2}$ mA. The experiments of [1] used a current $I \sim 10$ mA. This suggests that they were performed in the SCR. A typical value of inductance for ceramics is $L$ are bigger than $3$ [11]. As seen from Fig. 2, the value of $\alpha$ in the SCR for $1 < L < 5$ agrees very well with the experimental value.

References