Monte Carlo Study of Pseudo-Gap Temperature $T^\ast$ within JJA Model

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Abstract

We study pseudo-gap temperature $T^\ast$ of high-$T_c$ superconductors by a Monte Carlo simulation of anisotropic 3D Josephson Junction Array (JJA) model based on the Ginzburg-Landau theory. We investigate $T^\ast$ both in the cases of zero external current and finite external current $I$ in the JJA. It is found that, the external current $I$ depresses only a little the pseudo-gap temperature $T^\ast$, while the superconducting critical temperature $T_c$ is much affected by $I$.

Key words: Pseudo-gap temperature, High-$T_c$ cuprate superconductor, Josephson Junction Array model, Ginzburg-Landau theory

Much attention has been focused on the pseudo gap of high-$T_c$ cuprate superconductors. On the basis of the Josephson Junction Array (JJA) model for the high-$T_c$ cuprate superconductors [1–4], we have investigated the pseudo-gap temperature $T^\ast$ and the superconducting critical temperature $T_c$. In this paper, we report our result of the Monte Carlo simulation for the effect of the external current $I$ on $T^\ast$ and $T_c$.

We model the ceramic high-$T_c$ materials as a JJA which consists of weakly coupled superconducting grains on an anisotropic 3D lattice (i.e., a stack of 2D-lattice layers) [1–4]. The grain at the lattice site $i$ is characterized by the phase $\theta_i$ and the amplitude $|\phi_i|$ of the superconducting order parameter $\phi_i = |\phi_i| \exp(i\theta_i)$.

In previous papers [5,6], we performed the Monte Carlo simulation with a Hamiltonian in which only the phase $\theta_i$ was taken into account, and could investigate the effect of $I$ on $T_c$ only. In this paper, we investigate not only $T_c$ but also $T^\ast$ by considering both the phase $\theta_i$ and the amplitude $|\phi_i|$. Our effective Hamiltonian is given as

\[
F_{\text{eff}} = F(\{\theta_i, |\phi_i|\}) - T \sum_i \ln(|\phi_i|) \\
= F_0 + F_1 - T \sum_i \ln(|\phi_i|),
\]

where

\[
F_0 = - \sum_{i,j} |\phi_i|^2 \left[ \cos(\theta_i - \theta_j) - 1 \right] \\
- \alpha \sum_{i,k} |\phi_i|^2 \left[ \cos(\theta_i - \theta_k) - 1 \right] \\
- \sum_{i,j} I \cdot [\theta_i - \theta_j],
\]

\[
F_1 = \frac{1}{2} \sum_{i,j} (|\phi_i| - |\phi_j|)^2 \\
+ \frac{\alpha}{2} \sum_{i,k} (|\phi_i| - |\phi_k|)^2
\]

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Here, \( \sum_{i,j} \) means the summation over the neighboring intra sites in a 2D layer and \( \sum'_{i,k} \) over the neighboring inter sites between the layers. The parameter \( \alpha \) of the system anisotropy corresponds to \( \Gamma_2 \) of Ref. [7]; \( \alpha \to 1 \) (3D limit) and \( \alpha \to 0 \) (2D limit). The parameter \( A \) corresponds to \((a_\parallel/\xi_\parallel)^2\) and \( T_0 \) corresponds to \( T_{MF} \) [7].

We perform the Monte Carlo simulation on the 3D JJA system [Eqs. (1)–(3)] with \( A = 1 \) and the anisotropy ratio \( \alpha = 0.01 \). The system size is \( 20 \times 20 \times 20 \) with periodic boundary conditions. \( T_c \) is defined as the temperature at which the susceptibility \( \chi = \sum_{i,j} \langle \cos \theta_i \cos \theta_j \rangle \) diverges [5,6]. The symbol \( \langle \cdots \rangle \) represents the statistical average. \( T^* \) is defined as the temperature at which \( \sum_i \langle |\phi_i| \rangle = 0 \). In Table 1, we show the result obtained by the Monte Carlo Simulation. It is noticeable that, while the superconducting critical temperature \( T_c \) is much affected by \( I \) (namely, \( \sim 50\% \) decrease of \( T_c \)), the external current \( I \) depresses only a little the pseudo-gap temperature \( T^* \).

We hope that this result (i.e., the difference in the \( I \) sensitivity between \( T^* \) and \( T_c \)) can be observed experimentally by applying the external current to the high-\( T_c \) cuprate superconductors and simultaneously measuring the pseudo-gap temperature and the superconducting critical temperature. Such observations are expected to be helpful to identify the origin of the pseudo gap in the high-\( T_c \) cuprates, i.e., to identify whether or not the separation between the pseudo-gap temperature and the superconducting critical one is described by the anisotropic 3D JJA- and XY(phase)-model scenarios [1–8] for the superconductivity in the high-\( T_c \) cuprates.

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**References**


Table 1

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<th>( I )</th>
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<th>0.5</th>
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<td>0.26</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>( T_c )</td>
<td>0.09</td>
<td>0.09</td>
<td>0.05</td>
</tr>
</tbody>
</table>


